DETERMINING THE VALUE OF EMPLOYEE STOCK OPTIONS

Report Produced for the Ontario Teachers Pension Plan
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1. Background

It is now becoming increasingly accepted that companies should recognize the compensation their employees receive in the form of stock options as an expense on the income statement. One of the major difficulties in accounting for employee stock options in this way is the determination of the fair value of the options. Standard methods have been developed for valuing the options that trade on an exchange and in the over-the-counter market. However, there are a number of features of employee stock options that make it difficult to apply these standard methods to them.

We have been retained by the Ontario Teachers Pension Plan to examine how employee stock options should be valued. This report outlines our conclusions. It explains how FASB 123/CICA 3870 can be implemented and develops some enhancements to the basic methodology suggested in these accounting standards. In the interests of clarity, we focus on the most common type of employee stock option plan where a) the exercise price of an option remains constant during the option's life, b) the option can be exercised at any time during its life after an initial vesting period, and c) the employee cannot continue to hold the options after he or she has left the company. The approach we suggest can be adapted to value other options, for example those where the exercise price changes through time and those where the exercise price is linked to the value of a stock index.

Some spreadsheet-based software, ESOPVal, accompanies this report. This carries out the calculations necessary to implement our procedures and can be downloaded from www.rotman.utoronto.ca/~hull/ESOPS.

2. Standard Options Valuation Methods

Two alternatives for valuing equity options that trade on exchanges and in the over-the-counter market are the Black-Scholes approach and the binomial tree approach.

The Black-Scholes approach is appropriate for European options (that is, options that cannot be exercised prior to maturity). It is based on the pioneering work of Black, Scholes, and Merton published in 1973.[1] The valuation of a call option is expressed as a function of six parameters:

[1] See F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities" *Journal of Political Economy*, Vol 81 (May/June 1973), 637-659; and R. C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4 (Spring 1973), 141-183.

a) The stock price
b) The exercise price
c) The life of the option
d) The expected dividend yield
e) The risk-free rate of interest
f) The expected stock price volatility

The binomial tree approach is designed to value an American option (that is, an option that can be exercised at any time during its life). It is a numerical procedure proposed by Cox, Ross, and Rubinstein in 1979 and requires the same six parameters as Black-Scholes.[2] A tree representing possible future stock price movements is constructed and the option is valued by working back through the tree from the end to the beginning of the life of the option calculating the value of the option at each node of the tree. The American-style feature of the option is handled by testing whether it is optimal to exercise the option at each node of the tree. This involves setting the value of the option at the node equal to the greater of its value if exercised and its value if not exercised. The technical aspects of the Black-Scholes approach and the binomial tree approach are explained in Appendix A.

The Black-Scholes approach and the binomial tree approach make the same assumptions about the behaviour of the stock price.[3] The binomial tree approach can be used to value a European option by not testing for early exercise at each node of the tree. The binomial tree approach then gives the same price as the Black-Scholes approach.[4]

3. Nature of Employee Stock Options

As pointed out by authors such as Rubinstein (1995), employee stock options differ from the options that are traded on exchanges and in over-the-counter markets in a number of important respects:[5]

1. There is usually a vesting period of during which the options cannot be exercised. This vesting period can be as long as four years.
2. When employees leave their jobs they typically lose unvested options and any vested options that are out of the money. They have to exercise vested options that are in the money immediately.
3. Employees are not permitted to sell their employee stock options. They must exercise the options and sell the underlying shares in order to realize a cash benefit or diversify their portfolios. This tends to lead to employee stock options being exercised earlier than similar regular options.
4. There is some dilution when employee stock options are exercised.

[2] See J. Cox, S. Ross, and M. Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, Vol 7 (October 1979), 229-264.
[3] This assumption is known as geometric Brownian motion.
[4] There is a small numerical error in the result from the binomial tree approach. However, if 200 time steps are used for the tree, as in ESOPVal, this error is very small.
[5] See M. Rubinstein, "On the Accounting Valuation of Employee Stock Options", *Journal of Derivatives*, Vol. 3, No. 1 (Fall 1995), 8-24.

As a result the standard option valuation methods in Section 2 must be modified to handle employee stock options.

4. FASB 123/CICA Section 3870

The traditional way of accounting for stock options in the United States and Canada is the intrinsic value based method. In the United States this is based on Opinion 25 issued in 1972. Under this method the compensation cost of an employee stock option is assumed to be the excess, if any, of the market price of the stock over the exercise price on the date the option is granted. In the most common situation where options are granted with an exercise price equal to the current market price the intrinsic value method calculates the compensation cost as zero.

In October 1995 the Financial Accounting Standards Board published FASB 123 "Accounting for Stock Based Compensation". This statement encourages companies to adopt a fair value based method of accounting for stock options instead of the intrinsic value based method, but it does not require them to do so.[6] Following the publication of FASB 123 most companies continued to use the intrinsic value based method. Recently however, a number of companies such as Coca Cola and General Electric have adopted the fair value based method of accounting for stock options.

Appendix B of FASB 123 discusses the selection and use of an option-pricing model in some detail. Both the Black-Scholes model and the binomial tree model are considered to be acceptable by the Board. The input parameters used by FASB 123 are the ones that would be used to value a regular European or American option (see list in Section 2) except for the life of the option. When valuing a regular European option using the Black-Scholes model, the life of the option is set equal to the time until exercise. When valuing a regular American option using a binomial tree model, the life of the option is set equal to the time until the last allowable exercise date. FASB 123 sets the life of the option equal to what it terms the "expected life". This is the average time the option stays in existence assuming the employee does not leave during the vesting period. After valuing the option with this parameter for the option life, FASB 123 makes an adjustment to reflect the probability that the options might be forfeited during the vesting period, because the employee leaves the company.[7]

The basic FASB 123 procedure is best illustrated with an example. We will consider a company that grants the option shown in Table 1 to its employees.[8]

[6] Companies electing not to use the fair value based method must make pro forma disclosures of the effect of using the fair value based method.
[7] An alternative to making this adjustment is to assume that all options will vest and then later reverse the charge to income for those that do not vest.
[8] This is based on an example that appears in Appendix B of FASB 123.

Table 1: Sample Option	
Life of option	10 years
Vesting period	3 years
Stock price	$50
Exercise price	$50
Risk-free rate	7.5%
Expected volatility	30%
Expected dividend yield	2.5%

A normal 10-year European option would be valued using the Black-Scholes model at 20.47. A normal 10-year American option would be valued using a binomial tree model at 21.03. To value the employee stock option using the FASB procedure we first must estimate two additional parameters:

1. The employee exit rate during the vesting period. This is the probability that an employee will leave the company each year during the vesting period and is referred to as the forfeiture rate in FASB 123.[9]
2. The expected life of the option, i.e., average time the option stays in existence assuming the employee does not leave during the vesting period

Suppose that the employee exit rate is 3% per year and the expected life of the option is estimated as 6 years. Reducing the option life parameter in Black-Scholes from 10 years to 6 years reduces the value of the option value from 20.47 to 17.15. The employee exit rate of 3% means that on average 3% of this value is lost each year during the vesting period because employees leave the company. The FASB 123 value of the option is therefore be $0.97 \times 0.97 \times 0.97 \times 17.15 = 15.65$. If the binomial tree model were used the value of a six-year option would be calculated as 17.25 and the FASB 123 value of the option would be $0.97 \times 0.97 \times 0.97 \times 17.25 = 15.75$.

To determine the total value, the value of an individual option must be multiplied by the number of options of that type that are outstanding. Suppose that 100,000 options have been granted with the parameters in Table 1. Suppose further that the employee exit rate and expected life we have been using (3% and 6 years) are appropriate for all the options. The fair value of the options would be calculated as $100{,}000 \times 15.65 = \$1{,}565{,}000$ using the Black-Scholes model and $100{,}000 \times 15.75 = 1{,}575{,}000$ using the binomial tree model.[10]

In November 2001 Canadian Institute of Chartered Accountants (CICA) published Section 3870 of its Handbook dealing with stock-based compensation and other stock-

[9] We prefer the term employee exit rate because as we discuss later employees may leave the company after the vesting period. In this case they only forfeit their options if they are out of the money.

[10] In practice under FASB 123 options are divided into groups where the employee exit rate and expected life for the options in each group are considered to be approximately the same.

based payments. The valuation methods proposed in Section 3870 are essentially the same as those in FASB 123.

5. Implementation of Basic FASB 123/CICA 3870

FASB 123's use of the Black-Scholes and binomial tree models in conjunction with an "expected life" parameter does not have a solid theoretical basis. In Section 6 we will propose an enhancement to the basic FASB 123 approach that overcomes this weakness. For companies that prefer to implement the basic FASB 123/ CICA 3870 method without enhancements, this section examine key implementation issues and how they can be resolved.

FASB 123/CICA 3870 allows companies to use either the Black-Scholes model or the binomial tree model for calculating the fair value of an employee stock option. As mentioned earlier the Black-Scholes model is designed to value a plain vanilla European option and the binomial tree model (as it is usually used) is designed to value a plain vanilla American option. The binomial tree (American) price is always greater than or equal to the Black-Scholes (European) price, but the difference between the two is quite small for the dividend yields usually encountered in practice.[11] (For example, for the option in Table 1, the difference between the European and the American price is about 2% for a 10 year life and about 0.6% for a 6 year life.) In practice, therefore, when the FASB 123/CICA 3870 procedures are used, the choice between the Black-Scholes model and the binomial tree model usually makes very little difference. Most companies use the Black-Scholes model.

In addition to the parameters used to value a regular option, FASB 123/CICA 3870 requires estimates of

1. The employee exit rate during the vesting period
2. The expected life of the option

Employee exit rates can be estimated relatively easily from the turnover of employees in different categories. For example, if it is found that 5% of middle management employees leave (voluntarily or involuntarily) each year the employee exit rate should be set equal to 5% when valuing the options issued to middle management.

The expected life of a set of employee stock options is the length of time on average that options remains unexercised. Only options that vest should be considered for the purposes of calculating the average. This means that the expected life of an option must be at least as great as the time to vesting. It is likely that companies will be able to obtain only approximate estimates of the expected life. One approach is to observe how long on

[11] In the limit when the dividend yield is zero there is no difference at all between the American option price and the European option price.

average options have remained unexercised in the past.[12] Unfortunately the estimate obtained in this way is likely to depend on the period of time for which data is available. During periods when the market is rising fast options are likely to be exercised early and the expected life is likely to be low. During periods when the market is declining options are likely to remain unexercised for the whole of their life and the expected life is likely to be much longer.

An alternative approach suggested by FASB 123 in paragraph 282 is to use a model of how employees make their decisions on early exercise. This is probably a more robust approach. Suppose that we observe that employees voluntarily exercise options when the stock price is a multiple *M* of the exercise price for some value of *M*. We can modify the standard binomial tree to incorporate the following conditions:[13]

1. Options can be exercised only after the vesting period
2. An option is exercised at a node if the stock price at the node is at least M times the exercise price
3. There is a probability that the employee will leave the company during each short period of time after the vesting period. If the employee does so, the option is forfeited if it is out of the money and exercised immediately if it is in the money

Appendix B describes how the tree can be used to calculate the expected life of the option.

Table 2 shows the results of using this approach for the option in Table 1. The employee exit rate is assumed to be 3% throughout the life of the option. The first column of the table shows the value of *M* and the second column shows the expected life of the option when the employee uses an early exercise policy corresponding to this value of *M*.[14] The third column shows the Black-Scholes value of the option when the life of the option is set equal to the expected life. The Black-Scholes value in the table is adjusted for an employee exit rate of 3% per year during the vesting period as described in the previous section.

[12] In doing this calculation companies should consider both options that were exercised and those that were not. The average includes the time to exercise for the options that are exercised, and the life of the option for those that are not exercised.
[13] For technical reasons we use a trinomial tree rather than a binomial tree as discussed in Appendix B.
[14] Theoretically this is the option's expected life in a risk-neutral world.

Table 2: Dependence of the Value of the Option in Table 1 on Early Exercise Behaviour		
Value of *M*	Expected Life (yrs)	Black-Scholes Value
1.2	5.24	14.78
1.5	6.13	15.79
2.0	7.25	16.85
2.5	7.95	17.41
3.0	8.39	17.73

6. Enhanced FASB 123

In this section we examine the modifications to the standard option pricing models needed to make them capture the essential features of an employee stock option plan and suggest an improvement to the basic FASB 123 valuation procedure. We refer to this as Enhanced FASB 123. It conforms to the FASB 123 standard.[15] The general approach is similar to the basic valuation method in FASB 123 and has all the features FASB 123 requires a valuation model to have. The model is equivalent to the model that we describe in the previous section for estimating the expected life parameter.

In Section 3 we listed the four key features of employee stock options that make them different from a regular stock option. The impact of the dilution that occurs when employee stock options are exercised is usually very small and can reasonably be ignored. The key aspect of any valuation approach is how it deals with the other differences. These are:

1. There is usually a vesting period of during which the options cannot be exercised. This vesting period can be as long as four years.
2. When employees leave their jobs they typically lose unvested options and any vested options that are out of the money. They have to exercise vested options that are in the money immediately.
3. Employees are not permitted to sell their employee stock options. They must exercise the options and sell the underlying shares in order to realize a cash benefit or diversify their portfolios. This tends to lead to employee stock options being exercised earlier than similar regular options.

FASB 123 handles the first difference by estimating an employee exit rate during the vesting period. It can be argued that this rate is negatively correlated with the stock price because a) employees are more likely to be fired when the company is doing badly, and

[15] To quote from paragraph 154 of FASB 123: "The Board's intent in this Statement is for the …illustrations in Appendix B to be sufficiently broad that employers may adopt future refinements in the models that improve their application to employee stock options without requiring the Board to amend this statement."

b) employees are less likely to choose to leave a company voluntarily if their employee stock options are in the money. It is therefore likely that FASB 123's procedure for handling the first feature of employee stock options understates the value of the options. Estimating the negative correlation between employee exit rates and the stock price is difficult. Luckily however the impact of the correlation appears to be very low.[16]

FASB 123 handles the second and third feature by reducing the life of the option. The problem with this approach is that it is very difficult to estimate the reduction in life that is appropriate. FASB 123 suggests setting the life of the option equal to an estimate of the average time the options will remain unexercised. Although this may give reasonable results in many situations, it does not have a solid theoretical basis. The 'true' value of an option once it has vested is determined by a) the exercise strategy of the employee, and b) the possibility that the employee may be forced to exercise the option early or abandon it because he or she leaves the company. There is no reason why the FASB estimation procedure should produce a value close to the 'true' value.

The Enhanced FASB 123 model involves an extension of the binomial tree model. It differs from the basic FASB 123 model in that

1. It explicitly considers the possibility that the employee will leave the company after the vesting period. It requires the company to use employee turnover rates to estimate employee exit rates for both the pre-vesting period and the post-vesting period. [17]
2. It explicitly incorporates the employee's early exercise policy. It does this by assuming that early exercise happens when the stock price is a certain multiple, *M*, of the exercise price.

As explained in the previous section the binomial model can be extended to carry out calculation for the Enhanced FASB 123 model.[18] The exercise rules are

1. Options can be exercised only after the vesting period
2. An option is exercised if the stock price is at least *M* times the exercise price
3. There is a probability that the employee will leave the company during each short period of time after the vesting period. If the employee does so, the option is forfeited if it is out of the money and exercised immediately if it is in the money

Appendix B provides the technical details. Table 3 shows the price of the option in Table 1 for different assumptions about *M* and *e*, where *e* is the employee exit rate (assumed constant).

[16] In an example considered by Rubinstein, the use of a correlated forfeiture rate increases the option value from 30.75 to 31.63. See M. Rubinstein, "On the Accounting Valuation of Employee Stock Options", *Journal of Derivatives*, Vol. 3, No. 1 (Fall 1995), 8-24.

[17] Normally the pre-vesting and post-vesting employee exit rates will be the same. However, ESOPVal allows them to be different. By setting the pre-vesting employee exit rate in ESOPVal equal to zero the alternative accounting treatment in footnote 7 can be used.

[18] As mentioned in footnote 13, for technical reasons we use a trinomial rather than a binomial tree.

Table 3: Impact on Valuation of Option in Table 1 of Alternative Values of *M* and *e*.

The parameter, *M*, is the ratio of the stock price to the exercise price necessary to trigger voluntary early exercise. The parameter *e* is the employee exit rate (assumed to be the same pre-vesting and post-vesting).

	e = 3%	*e* = 5%	*e* = 7%	*e* = 10%
M = 1.2	13.13	12.28	11.47	10.33
M = 1.5	15.13	14.06	13.07	11.69
M = 2.0	17.09	15.80	14.61	12.97
M = 2.5	17.97	16.57	15.28	13.53
M = 3.0	18.34	16.89	15.56	13.75

7. Empirical Evidence

There are relatively few statistics available on the actual exercise behaviour of employees in different types of companies to assist in choosing the FASB 123/CICA 3870 expected life parameter and to assist in choosing *M* in the Enhanced FASB 123 model. Huddart and Lang (1995) and Carpenter (1998) provide some results.[19] Carpenter looked at a sample of option exercises by top executives at 40 firms between 1979 and 1994. All the options had 10-year lives. The average vesting period was 1.96 years; the average time of exercise was 5.83 years; and the stock price at the time of exercise was 2.8 times the exercise price. Huddart and Lang looked at five firms, but considered all employees not just top executives. They found that the average time of exercise was 3.4 years and the average ratio of the stock price to the exercise price at the time of exercise was 2.2.

Unfortunately these results cannot be used in a direct way to estimate the expected life parameter for FASB 123/CICA 3870. Unless it happens that all options are eventually exercised, the average time options remain unexercised (which is what we are interested in estimating) will be greater than the average time to exercise. The average ratio of the stock price to the exercise price at the time of exercise is only an approximate estimate of an employee's exercise policy. This is because at the end of the vesting period the stock price might be well above the minimum necessary to trigger exercise. Also at the end of the life of an option, exercise will take place for all stock prices above the exercise price.

The average time to exercise and the ratio of the stock price to the strike price at the time of exercise is lower for the Huddart and Lang sample than for the Carpenter sample. This suggests that top executives may wait longer than more junior employees before

[19] See Huddart S. and M. Lang, "Employee stock option exercises: An empirical analysis," *Journal of Accounting and Economics*, 21 (1996), 5-43; J. Carpenter, "The exercise and valuation of executive stock options," *Journal of Financial Economics*, 48 (1998), 127-158.

exercising. We can conjecture that this may be because they have less need to exercise options for personal liquidity reasons.

8. Estimating Parameters

In this section we discuss how the risk-free rate, the expected dividend yield, and the volatility can be estimated. The estimation of other parameters has already been discussed.

FASB 123 states that the risk-free rate should be the rate obtained on zero-coupon government bonds with maturity equal to the expected life of the option.[20] In theory the rate should be measured with continuous compounding, but in practice using semiannual or annual compounding makes very little difference. We recommend assuming that the expected life is two thirds of the maximum life for the purposes of estimating the risk-free rate.

Dividend yield should be estimated as the expected average dividend yield during the expected life of the option. The dividend yield should be measured with continuous compounding, but in practice using quarterly, semiannual or annual compounding makes very little difference. One guide as to the dividend yield likely in the future is the average dividend yield received by shareholders in the last two or three years. However, adjustments are necessary if material changes to the company's dividend policy are anticipated. A company that does not pay dividends and has no plans to do so should use a dividend yield of zero when valuing employee stock options.

The volatility used should be the expected volatility per year during the expected life of the option. Sometimes this can be estimated from the implied volatilities of long-dated options on the stock that are traded in the over-the-counter market. Usually it is necessary to estimate it from historical data.

We now explain how historical data is used to estimate volatility. If possible, the length of the period covered by the historical data used should be approximately equal to the expected life of the option. Suppose that we have n+1 observations of the stock price: S_0, S_1, S_2,..,S_n. We first use this data to calculate the following n numbers: $\ln(S_1/S_0)$, $\ln(S_2/S_1)$, $\ln(S_3/S_2)$, ….,$\ln(S_n/S_{n-1})$. The next stage is to calculate the standard deviation of these n numbers. Denote this standard deviation by u. If our historical data consists of observations at monthly intervals on the stock price, u is the monthly volatility and the volatility per year is $u\sqrt{12}$; if it consists of observations at weekly intervals (as recommended by FASB 123), u is the weekly volatility and the volatility per year is $u\sqrt{52}$; if it consists of observations at daily intervals, u is the daily volatility and the volatility per year is $u\sqrt{252}$. (The latter reflects the fact that there are 252 trading days per year.)

[20] An argument can be made for using the LIBOR/swap rate instead of the Treasury rate. This is what most investment banks do when valuing options.

9. The Software

The software that accompanies this report consists of three Excel worksheets:

1. Basic FASB 123
2. Enhanced FASB 123
3. Both models

The spreadsheets can be used to value an employee stock option either on the grant date or at a later date. The third worksheet has been included for convenience. It shows the contents of the first and second worksheet together.

Basic FASB 123 allows the value of the option to be calculated based on the stock price, the exercise price, the expected life, the risk-free rate, the expected volatility, and the expected dividend yield, the vesting period and the employee exit rate during the vesting period. The value can be calculated using either the Black-Scholes model or the binomial tree model. The expected life can be estimated directly or calculated from the Enhanced FASB 123 worksheet. The technical details of the calculations are in Appendix A.

Enhanced FASB 123 calculates the value of the option and its expected life as indicated in Section 6 based on the stock price, the exercise price, the maximum time to exercise, the risk-free rate, the expected volatility, the expected dividend yield, the remaining vesting period, the employee exit rate during the vesting period, the employee exit rate after the vesting period, and the early exercise multiple (i.e., ratio of the stock price to the exercise price necessary to trigger voluntary early exercise.) The technical details of the calculations are in Appendix B

Once all the data has been entered, the user should click outside the data entry cells and then click on the calculate button. The results of the calculations will then be displayed.

Both worksheets use 200 time steps when the binomial or trinomial tree approach is used. In either the Basic FASB 123 worksheet or the Enhanced FASB 123 worksheet, the user can set the employee exit rate during the vesting period to zero. This is appropriate if the user plans to assume that all options vest and then reverse the expense for those that do not in fact vest. (See footnote 7)

10. Conclusions

This report has discussed practical issues in valuing an employee stock option for FASB 123 and CICA 3870. The main difficulty in implementing the basic FASB 123/CICA 3870 valuation method lies in estimating the expected life of the option. One approach is to look at the average time different categories of employee stock options have remained unexercised in the past. Another approach is to use a model of the exercise behaviour of employees.

The basic FASB 123 method uses the expected life of the option as an input. Option pricing models are constructed so that the correct input is the total potential life of the option (i.e., the maximum time that can elapse until the option is exercised). We have proposed a modification to the basic FASB 123 that overcomes this weakness. The modification is very much in the spirit of the approaches recommended in FASB 123.

APPENDIX A
Black-Scholes and Binomial Tree Models

Define

S	Stock price
K	Exercise price
T	Remaining life of option (i.e. maximum time that can elapse until exercise)
R	Risk-free rate
σ	Expected volatility
Q	Expected dividend yield

The Black-Scholes model is designed to value a European option (i.e. an option that can only be exercised on one date. The formula for the price of a call option is:

$$Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

The binomial tree model is designed to value an American option; that is, an option that can be exercised any time during its life. In the binomial tree model we divide the life of the option into N time intervals. Define:

$$\delta t = \frac{T}{N}$$

$$u = e^{\sigma\sqrt{\delta t}}$$

$$d = \frac{1}{u}$$

$$a = e^{(r-q)\delta t}$$

$$p = \frac{a - d}{u - d}$$

At the end of the ith time interval we consider i+1 stock prices. We denote these by $S_{i,0}$, $S_{i,1}$, …., $S_{i,i}$. This means that there is one stock price $S_{0,0}$ today (this equals S); there are two stock prices $S_{1,0}$ an $S_{1,1}$ at time δt; there are three stock prices $S_{2,0}$, $S_{2,1}$, and $S_{2,2}$ at time $2\delta t$ and so on. We define $f_{i,j}$ as the value of the option price at the end of the ith interval when the stock price is $S_{i,j}$. The equations for $S_{i,j}$ and $f_{i,j}$ are:

$$S_{i,j} = u^j d^{i-j}$$

$$f_{N,j} = \max(S_{N,j} - K, 0)$$

and

$$f_{i,j} = \max\{S_{i,j} - K, e^{-r\,\delta t}[\,pf_{i+1,j+1} + (1-p)\,f_{i+1,j}\,]\} \text{ when } 0 \le i \le N-1$$

The value of the option is $f_{0,0}$.

Under FASB 123 the value of the option is calculated by a) setting the option life equal to the expected life, b) calculating the value of the option using either the Black-Scholes or the binomial tree model, and c) multiplying the result by $(1\text{-}e_1)^v$ where e_1 is the employee exit rate during the vesting period and v is the time in years until the end of the vesting period.

APPENDIX B
Implementation of Enhanced FASB 123 and Calculation of Expected Life for Basic FASB 123

This appendix describes how the binomial tree model can be used to implement the enhanced FASB 123 method and to estimate the expected life of the options that vest in the basic FASB 123 method. We use the same notation as Appendix A. The user must estimate the early exercise multiple percentage (i.e., the ratio of the stock price to the exercise price necessary to trigger voluntary exercise by the employee.). Suppose this is *M* so that the option is exercised when the stock price is greater than or equal to *KM*. Define v as the time until the options vest, e_1 as the employee exit rate during the vesting period, and e_2 as the employee exit rate after the vesting period.

Define $e_2^* = \ln(1+e_2)$. This is a small adjustment to allow for the fact that after the vesting period we consider the possibility of an employee leaving in short periods of time of length δt rather than in periods of time of length 1 year.

The rules for calculating $f_{i,j}$ in the binomial tree method described in Appendix A are modified to:

$$f_{N,j} = \max(S_{N,j} - K, 0)$$

When $0 \le i \le N-1$:

If $i\delta t > v$ and $S_{i,j} > KM$ then $f_{i,j} = S_{i,j} - K$

If $i\delta t > v$ and $S_{i,j} < KM$ then $f_{i,j} = (1 - w_2^* \delta t) e^{-r\delta t} [p f_{i+1,j+1} + (1-p) f_{i+1,j}] + w_2^* \delta t \max(S_{i,j} - K, 0)$

If $i\delta t < v$ then $f_{i,j} = e^{-r\delta t} [p f_{i+1,j+1} + (1-p) f_{i+1,j}]$

The value of the option is

$$f_{0,0}(1 - w_1)^v$$

Define $L_{i,j}$ as the expected life of the option at time $i\delta t$ when the stock price is $S_{i,j}$. We set:

$$L_{N,j} = 0$$

When $0 \le i \le N-1$:

If $i\delta t \ge v$ and $S_{i,j} \ge KM$ then $L_{i,j} = 0$

If $i\delta t \ge v$ and $S_{i,j} < KM$ then $L_{i,j} = (1 - w_2^* \delta t)[p L_{i+1,j+1} + (1-p) L_{i+1,j} + \delta t]$

If $i\delta t < v$ then $L_{i,j} = p L_{i+1,j+1} + (1-p) L_{i+1,j} + \delta t$

The option we are valuing is an example of what is analysts term a barrier option. To obtain more accuracy it is preferable to use a trinomial tree rather than a binomial and to

adjust parameters so that at any given time $i\delta t$ there is a node at the point where the stock price equals KM.[21] This is what ESPVal does.

[21] For details on how this can be done see J.C. Hull "Options, Futures, and Other Derivatives" fifth edition, 2002, Prentice Hall.